MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2017/2018

DIM5058 – MATHEMATICAL TECHNIQUES 1

(For DIT students only)

27 OCTOBER 2017 09.00 am - 11.00 am (2 Hours)

INSTRUCTIONS TO STUDENT

- 1. This question paper consists of 3 pages with 4 questions. Key formulae are given in the Appendix.
- 2. Answer ALL questions.
- 3. Write your answers in the answer booklet provided.
- 4. All necessary working steps must be shown.

Question 1

- a) Rationalize the denominator of $\frac{\sqrt{5}+5}{1-\sqrt{5}}$ and simplify your answer. (4 marks)
- b) Solve the equation $3xy^2 75x 2y^2 = -50$. (5 marks)
- c) Solve the equation $5x^2 + 2x 7 = 0$ by using completing the square method. (6 marks)
- d) Solve the absolute inequality |8x-5| + 2x 1 < 10 + 2x and represent the answer on an interval notation. (5 marks)

[TOTAL 20 MARKS]

Question 2

- a) Given that $g(x) = \frac{3x-5}{4}$ and $h(x) = \frac{4+3x}{9}$.
 - i) Find (g+h)(x). (4 marks)
 - ii) Evaluate $g^{-1}(1)$. (3 marks)
- b) Given a quadratic function f(x) = (x+2)(1-x).
 - i) Determine whether the parabola opens upward or downward. (2 marks)
 - ii) Find the vertex of the parabola.

(3 marks)

iii) Find the x-intercept(s).

(3 marks)

iv) Find the y – intercept.

- (1 marks) (4 marks)
- v) Sketch the parabola and label the necessary points.

[TOTAL 20 MARKS]

Continued...

Question 3

a) Given that
$$A = \begin{bmatrix} 3 & -7 & 4 \\ 4 & 5 & 8 \\ 8 & -6 & 10 \end{bmatrix}$$
 and $B = \begin{bmatrix} 1 & 10 & -7 \\ 3 & 1 & -9 \\ -2 & 11 & 1 \end{bmatrix}$.

- i) Find 3A + 4B. (4 marks)
- ii) Find $B^T A$. (3 marks)
- b) Given that $A = \begin{bmatrix} 5 & 4 \\ 4 & 3 \end{bmatrix}$ and $M = \begin{bmatrix} -14 & 20 \\ -12 & 17 \end{bmatrix}$.

i) If
$$AA^{-1} = A^{-1}A = I$$
, find A^{-1} . (3 marks)

ii) From part (i), show that
$$A^{-1}M^{T} = \begin{bmatrix} 122 & 104 \\ -156 & -133 \end{bmatrix}$$
. (5 marks)

c) A furniture company produces three types of desks: a traditional model, modern model and deluxe model. Each desk is manufactured in three stages: cutting, construction and finishing. The time requirements for each model and manufacturing stage are given in the following table.

Manufactured		Model	
Process	Traditional	Modern	Deluxe
Cutting	2	3	2
Construction	2	1	3
Finishing	1	1	2

Each week the company has available a maximum of 100 hours for cutting, 100 hours for construction and 65 hours for finishing. [Hint: Let x, y and z be the number of desks for traditional, modern and deluxe model respectively.]

- i) Represent the above information in the form of AX = B. (2 marks)
- ii) From part (i), solve the value of x, y and z by using **Cramer's Rule**. (13 marks)

[TOTAL 30 MARKS]

Continued...

Question 4

- a) Write the first five terms of the sequence whose general form is $a_n = \frac{3a_{n-1} + 1}{(n+1)!}$ and given the first term is $a_1 = -5$. (4 marks)
- b) Find the sum for $\sum_{x=1}^{6} \frac{(2x+5)^2}{2^x}$. (4 marks)
- c) Insert three arithmetic mean between number 13 and 161. (3 marks)
- d) At the corner section of a football stadium, there are 8 seats in the first row and 35 rows in total. Each successive row contains 4 additional seats.
 - i) Find the first term and the common difference. (2 marks)
 - ii) Calculate the number of seats in the last row. (2 marks)
 - iii) Find the total number of seats in the corner section. (2 marks)
- e) Sarah has been hired by PQR company with an annual salary of RM28,000 and is expected to receive an annual increase of 6%. What will Sarah's annual salary be in her ninth year of service? (3 marks)
- f) The sum to infinity of a geometric series is 280. If the first term is 182, find the common ratio. (4 marks)
- g) Find the 4th and 9th term for the binomial expansion $(2x^3 + y^2)^{10}$. (6 marks)

[TOTAL 30 MARKS]

APPENDIX – KEY FORMULA

Inequalities:

Inequality	Solution
x < a	-a < x < a
$ x \le a$	$-a \le x \le a$
x > a	x < -a or $x > a$
$ x \ge a$	$x \le -a$ or $x \ge a$

Completing the square: $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

Quadratic formula: If $ax^2 + bx + c = 0$ where $a \ne 0$, then, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Standard form of a quadratic function: $f(x) = a(x - h)^2 + k$, $a \ne 0$

Determinant of a 2 × 2 matrix	Determinant of a 3 × 3 matrix
$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$	$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ $= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$
Inverse of a 2×2 matrix	Inverse of a 3 × 3 matrix
If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $ad - bc \neq 0$.	$A^{-1} = \frac{1}{ A } \begin{bmatrix} c_{ij} \end{bmatrix}^T$ $A^{-1} = \frac{1}{ A } \begin{bmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{23} & c_{33} \end{bmatrix}$ $A^{-1} = \frac{1}{ A } adj A$ where $\begin{bmatrix} c_{ij} \end{bmatrix}^T$ is called the adjoint of A (adj A). c_{ij} of the entry $a_{ij} = (-1)^{i+j} M_{ij}$

Cramer's Rule for 2 × 2 matrix	Cramer's Rule for 3 × 3 matrix
If $a_1x + b_1y = c_1$ $a_2x + b_2y = c_2$	$a_1x + b_1y + c_1z = d_1$ If $a_2x + b_2y + c_2z = d_2$ $a_3x + b_3y + c_3z = d_3$
then $x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \\ \hline \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} $ and $y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \\ \hline \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$,	then $x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$, $z = \frac{D_z}{D}$ where $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix},$
where $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0$	$D_{y} = \begin{vmatrix} a_{1} & d_{1} & c_{1} \\ a_{2} & d_{2} & c_{2} \\ a_{3} & d_{3} & c_{3} \end{vmatrix}, D_{z} = \begin{vmatrix} a_{1} & b_{1} & d_{1} \\ a_{2} & b_{2} & d_{2} \\ a_{3} & b_{3} & d_{3} \end{vmatrix}$

Arithmetic sequence	Geometric sequence
$a_n = a_1 + (n-1)d$	$a_n = a_1 r^{n-1}, S_n = \frac{a_1 (1 - r^n)}{1 - r}$
$S_n = \frac{n}{2} (a_1 + a_n)$	$S_{\infty} = \frac{a_1}{1-r}, r < 1$

Binomial Theorem

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k; \quad n \ge 1$$

The $(r+1)^{st}$ term in the expansion of $(a+b)^n$ is $\binom{n}{r}a^{n-r}b^r$.